REFINED SIMULATIONS OF VIOLENT SLOSHING FLOWS BY AN ENHANCED PARTICLE METHOD

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The paper presents enhanced simulations of violent sloshing flows by an improved Moving Particle Semi-implicit (MPS) method which benefits from four improvements, namely, a corrected pressure gradient model, a higher order source term, a higher order Laplacian model and a newly proposed modified Poisson Pressure Equation (PPE) consisting of a high-order main term and two error compensating terms with dynamic coefficients as functions of the instantaneous flow field. Enhanced performance of the refined MPS method is demonstrated through the simulations of violent sloshing flows induced by sway excitations and rotational ones.

Key Words : *Violent Sloshing, impact pressure, particle method, MPS, unphysical pressure fluctuations*

1. INTRODUCTION

The motion of a liquid inside a container, or the so-called sloshing phenomenon, is of considerable importance in coastal and offshore engineering considering the safety of sea transport of oil and Liquefied Natural Gas (LNG). Under relatively large-amplitude external excitations, the liquid inside a partially filled container may experience violent motions, breaking and fragmentations, resulting in large localized impact pressures.

Conventional numerical methods, namely, the Eulerian grid-based methods, often face several limitations and complications when dealing with violent fluid flows accompanied by breaking and fragmentations. On the other hand, recent interest has been focused on the new generation computational methods, namely, the particle methods. Due to their gridless feature particle methods are inherently well-suited to analyze problems involving large deformations and fluid fragmentations. Further, because of their Lagrangian nature, particle methods are free of numerical diffusion corresponding to advection terms often seen as a major shortcoming of Eulerian methods. Hence, particle methods appear to be appropriate candidates for analysis of violent sloshing flows. Nevertheless, due to applications of simplified, incomplete and inconsistent numerical schemes, original versions of particle methods have a major common shortcoming related to existence of unphysical pressure oscillations (Gotoh et al., 2005; Gotoh, 2009; Khayyer and Gotoh, 2009a,b).

This study presents refined simulations of violent sloshing flows by an improved particle method, to be more specific, an enhanced MPS (Moving Particle Semi-implicit) method (Koshizuka and Oka, 1996). The enhanced MPS method benefits from four improvements, namely, a Corrected gradient model (Khayyer and Gotoh, 2008), a Higher order Source term of Poisson Pressure Equation (PPE) (Khayyer and Gotoh, 2009a), a Higher order Laplacian model of the PPE (Khayyer and Gotoh, 2010) (CMPS-HS-HL) and a new numerical scheme proposed for enhancement of volume conservation as well as a more accurate projection. Two cases of sloshing waves induced by sway excitations (Kishev et al., 2006) and rotational ones (Delorme et al., 2009) are considered to confirm the enhanced performance of the improved MPS method.

2. A MULTI-TERM SOURCE OF POISSON PRESSURE EQUATION

To enhance the accuracy of numerical solutions, i.e. to obtain instantaneous divergence free velocity fields, overall volume conservation and accurate/stabilized pressure/velocity fields, we may introduce some Error-Compensating terms in the Source term of PPE (abbreviated as ECS). The error-compensating terms should be measures for both instantaneous and accumulative violations of fluid incompressibility. Thus, the modified PPE is introduced as:

$$\frac{\Delta t}{\rho}\left(\nabla^2 p_{k+1}\right)_i = \frac{1}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)_i^* + \mathrm{func}\left[\frac{1}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)_i^k , \frac{1}{\Delta t}\frac{n^k - n_0}{n_0}\right] \tag{1}$$

The first error-compensating parameter in Eq. 1 corresponds to the instantaneous time variation of particle number density at time step k or the divergence of velocity at this time step (which both should be zero theoretically). The second parameter reflects the deviation of n at time step k from the constant n_0 or the time rate of overall volumetric change at time step k, i.e., it accounts for the accumulative error in particle number density.

Here we consider a linear summation form of the error-compensating parameters introduced in Eq. 1. The coefficients of the error compensating parameters are proposed on the basis of instantaneous flow features and high-order numerical schemes are applied for both the main term as well as the error compensating parts of the multi-term PPE. Finally, the effect of free-surface is considered in derivation of the multi-term PPE. The multi-term PPE is proposed as:

$$\left(\frac{\Delta t}{\rho}\nabla^2 p_{k+1}\right)_i = \frac{1}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)_i^* + \left|\frac{n^k - n_i}{n_i}\right|\left[\frac{1}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)_i^k\right] + \left|\frac{\Delta t}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)_i^k\right|\left[\frac{1}{\Delta t}\frac{n^k - n_i}{n_i}\right] \tag{2}$$

In Eq. 2, n_i corresponds to an initial particle number density of a target particle i. This would be equal to n_0, the initial constant particle number density, if the target particle is an inner fluid particle. For particles at and close to the free-surface the initially calculated n_i values would be considerably smaller than n_0 which is calculated based on the initial arrangement of (equally spaced) inner particles. In case of violent fluid flows, one particle close to the free-surface may not remain close to that region during the calculation. Thus, n_i needs to be updated every N (=20) time steps.

Focusing on Eq. 2, it is clear that both error-compensating functions as well as their coefficients converge to zero when the incompressibility of fluid is numerically satisfied. Further, when the trend of the variations of n is towards n_0, the error compensating terms would cancel out each other and the source term would be calculated solely on the basis of calculated instantaneous time variations of n at the prediction step of time step k+1 (or pseudo time step k+1/2).

From Eq. 2, it is also evident that we are dealing with velocity of particle number density variations as well as its deviations (distance) from the constant initial one (n_0). It is also worth to mention here that that an equivalent form of Eq. 2 can be derived for other projection-based particle methods including the Incompressible SPH (ISPH) method.

3. SLOSHING SIMULATIONS

(1) sloshing flows by sway excitations

Violent sloshing flows induced by sway-type excitations are simulated. Conditions of the simulations correspond to the experiment by Kishev et al. (2006). **Fig. 1** shows a schematic sketch of the calculation domain as well as the simulation conditions. Point A indicates the pressure measuring point.

Fig. 2 presents a qualitative comparison in between the experimental photos and their corresponding snapshots obtained by the CMPS-HS-HL-ECS method. These snapshots appear to be in a good qualitative agreement with the experiment as the impact of the jet on the tank's roof and the reversing jet are well reproduced.

Fig. 3 shows the time history of pressure at measuring point A corresponding to Case B (Fig. 1). From this figure, it is clear that the results by both CMPS-HS and CMPS-HS-HL methods contain frequent and large-amplitude unphysical oscillations that would make these two methods practically inapplicable to predict the wave impact pressure in a violent sloshing flow. Application of the ECS terms, however, has resulted in a significantly improved pressure trace and better approximations of the peak pressures.

(2) sloshing flows by rotational excitations

A violent sloshing flow induced by a rotational excitation is simulated to further investigate the applicability of CMPS-HS-HL-ECS method for prediction of sloshing-induced impact pressure. The simulation condition is set equivalent to that in the experiment by Delorme et al. (2009). **Fig. 4** shows a schematic sketch of the computational domain as well as the simulation conditions.

Fig. 5 presents two typical snapshots illustrating the formation of a plunging-type breaking and its violent impact on the tank's wall.

Fig. 6(a) depicts time variations of pressure at measuring point B, by the CMPS-HS-HL-ECS method. The results by an improved SPH model,

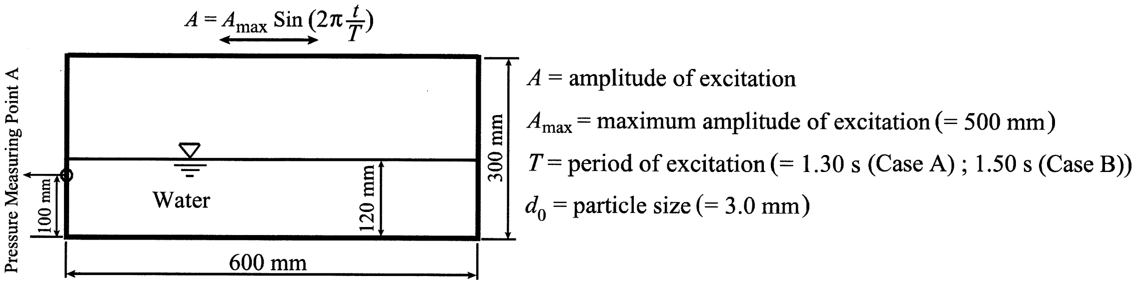

Fig. 1 A violent sloshing flow induced by sway excitations (Kishev et al., 2006) - Schematic sketch of calculation domain

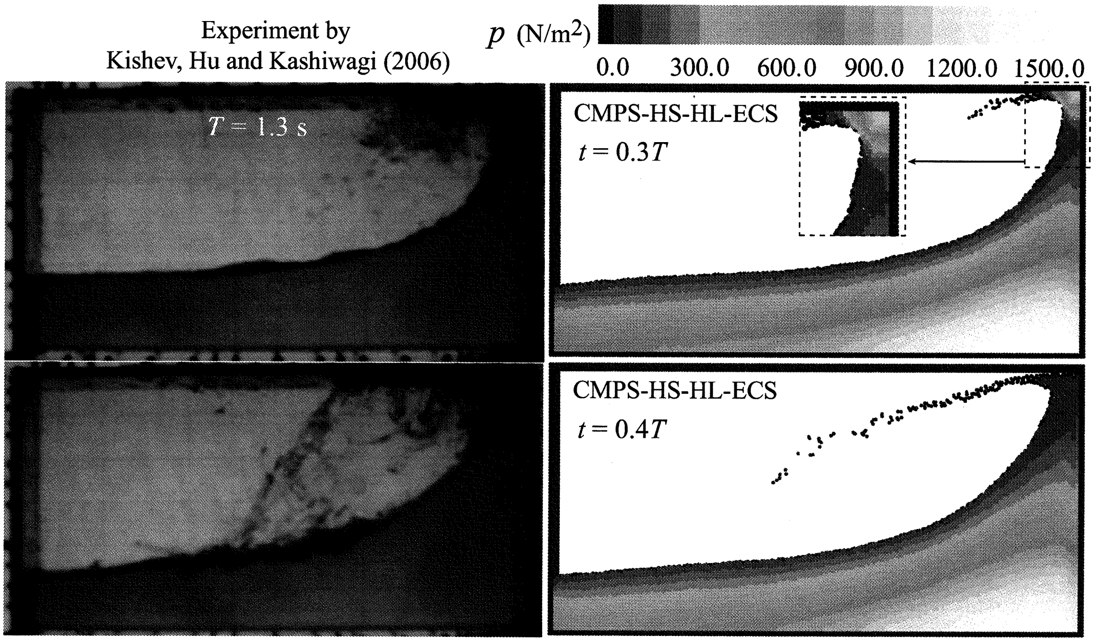

Fig. 2 A violent sloshing flow induced by sway excitations - Qualitative experiment-simulation comparison (Case A)

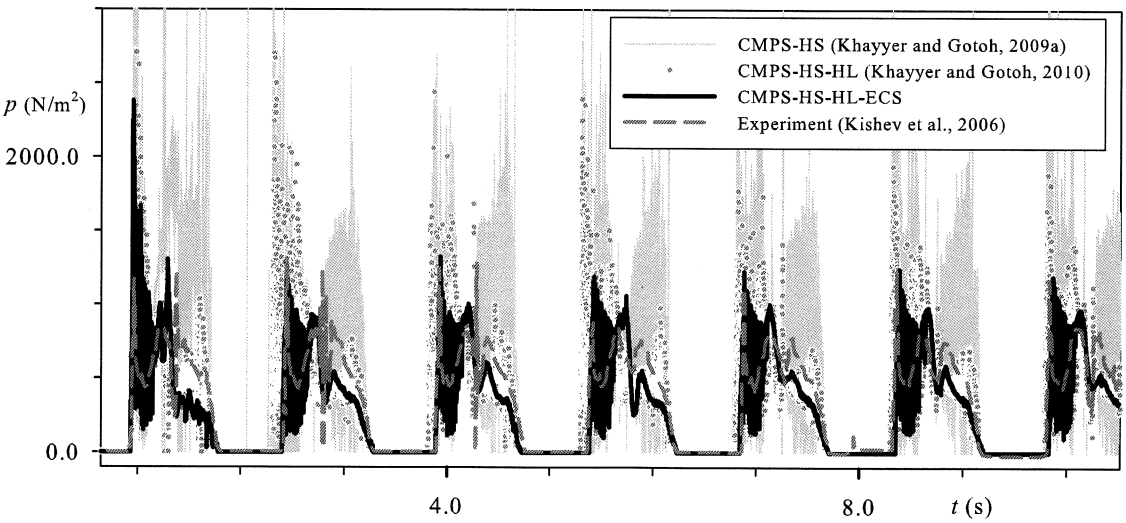

Fig.3 A violent sloshing flow induced by sway excitations - Time histories of calculated pressure at measuring point A (Case B)

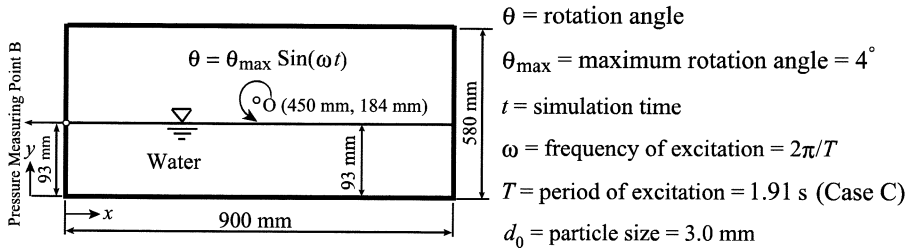

Fig.4 A violent sloshing flow induced by rotational excitations (Delorme et al., 2009) - Schematic sketch of calculation domain

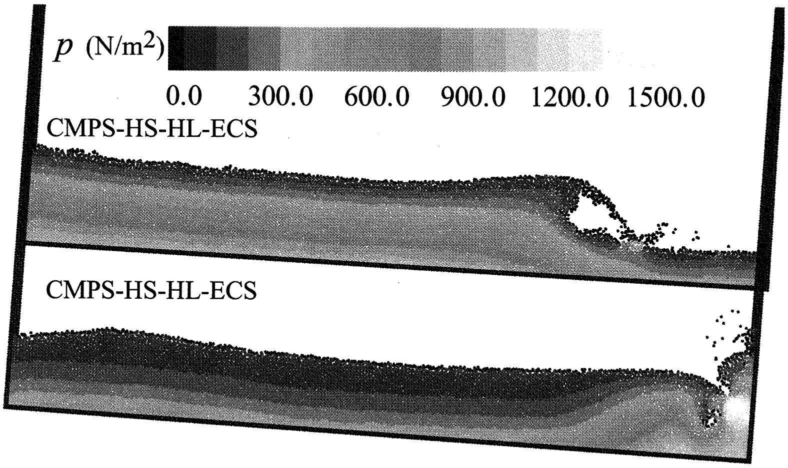

Fig.5 A violent sloshing flow - Two typical snapshots illustrating a plunging-type breaking and its impact on the tank's wall (Case C)

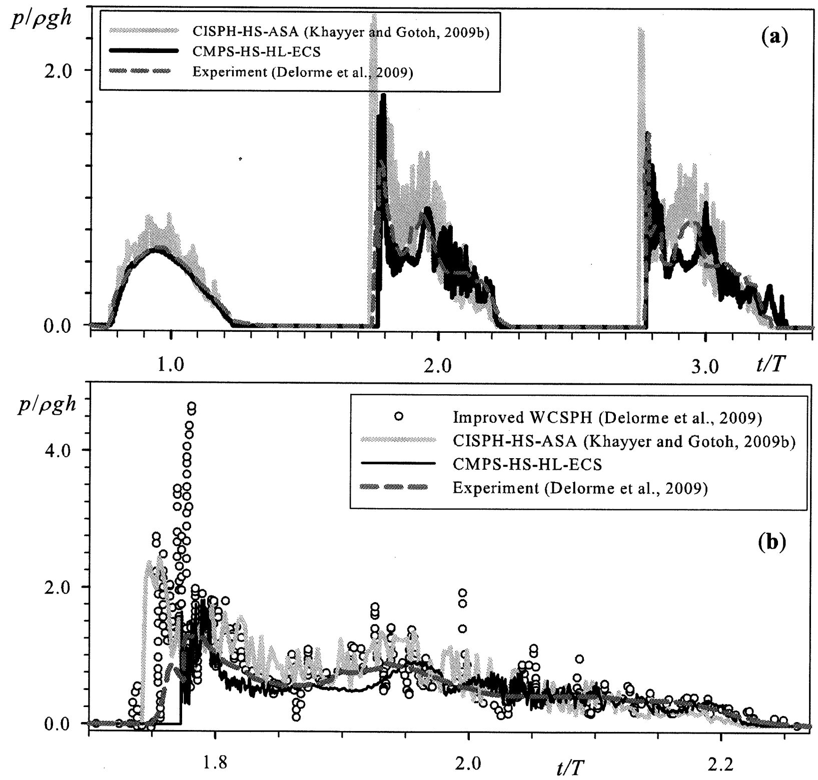

Fig.6 Time histories of calculated pressure by improved MPS, improved ISPH (a) and improved WCSPH (b) at measuring point B

namely, CISPH-HS-ASA (Khayyer and Gotoh, 2009b) benefiting from three improvements are also included for comparison. From this figure, the CMPS-HS-HL-ECS has resulted in a relatively accurate and less-fluctuating pressure trace. To further illustrate the enhanced performance of the newly proposed method, the results by an improved Weakly Compressible SPH (WCSPH; Delorme et al., 2009) method are shown in **Fig. 6(b)**. In comparison with the improved SPH methods, the CMPS-HS-HL-ECS method has provided a refined pressure calculation with a better prediction of the peak pressure.

4. CONCLUDING REMARKS

Refined simulations of violent sloshing flows are presented by an enhanced particle method characterized by three previously proposed improvements (Khayyer and Gotoh, 2008, 2009a, 2010) as well as a newly introduced one corresponding to a modified source term of the Poisson Pressure Equation (PPE). The modified source term comprises of a high-order main term and two high-order error-compensating terms with dynamic coefficients as functions of instantaneous flow field. Enhanced performance of the refined particle method is demonstrated through the simulation of violent sloshing flows induced by sway excitations and rotational ones.

The numerical results of this paper are obtained by a single-phase particle-based model. Further refined simulations of violent sloshing flows are expected to be obtained by a two-phase particle method developed in a mathematically consistent, physically sound and computationally efficient framework. Development of a precise SPS (Sub-Particle-Scale) turbulence model (Gotoh et al., 2001) is another key issue for accurate and reliable simulations of violent sloshing flows by particle methods. The papers by Gotoh and Sakai (2006) and Gotoh (2009) highlight the key issues for accurate and reliable particle-based simulations of hydrodynamic fluid flows.

REFERENCES

Delorme, L., A. Colagrossi, A. Souto-Iglesias, R. Zamora-Rodriguez, and E. Botia-Vera (2009): A set of canonical problems in sloshing, Part I: Pressure field in forced roll-comparison between experimental results and SPH. *Ocean Eng.*, 36(2), 168-178.

Gotoh, H. (2009): Lagrangian Particle Method as Advanced Technology for Numerical Wave Flume. *International Journal of Offshore and Polar Engineering*, 19(3), 161-167.

Gotoh, H., H. Ikari, T. Memita and T. Sakai (2005): Lagrangian particle method for simulation of wave overtopping on a vertical seawall. *Coast. Eng. J.*, 47, 157-181.

Gotoh, H. and T. Sakai (2006): Key issues in the particle method for computation of wave breaking, *Coastal Engineering*, 53, 171-179.

Gotoh, H., T. Shibahara and T. Sakai (2001): Sub-Particle-Scale Turbulence Model for the MPS Method -Lagrangian Flow Model for Hydraulic Engineering, *Comput. Fluid Dyn. J.*, 9(4), 339-347.

Khayyer, A. and H. Gotoh (2008): Development of CMPS method for accurate water-surface tracking in breaking waves. *Coast. Eng. J.*, 50(2), 179-207.

Khayyer, A. and H. Gotoh (2009a): Modified Moving Particle Semi-implicit methods for the prediction of 2D wave impact pressure. *Coastal Eng.*, 56, 419-440.

Khayyer, A. and H. Gotoh (2009b): Wave impact pressure calculations by improved SPH methods. *International Journal of Offshore and Polar Engineering*, 19(4), 300-307.

Khayyer, A. and H. Gotoh (2010): A Higher Order Laplacian Model for Enhancement and Stabilization of Pressure Calculation by the MPS Method. *Applied Ocean Res.*, 32(1): 124-131.

Kishev, Z. R., C. Hu and M. Kashiwagi (2006): Numerical simulation of violent sloshing by a CIP-based method. *Journal of Marine Science and Technology*, 11, 111-122.

Koshizuka, S. and Y. Oka (1996): Moving particle semi-implicit method for fragmentation of incompressible fluid. *Nuclear Science and Engineering*, 123, 421-434.

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